Astronomy preparation
Celestial mechanics – Basic problems 1 – Solutions

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Problem 1

The simplest way to obtain the value of gravitational acceleration is to use Newton's Second Law:

$$F = ma = mg = G\frac{mM}{r^2} \quad \rightarrow \quad g = \frac{GM}{r^2}.$$

Substituting the masses and radii of each celestial body (⊙ is a symbol commonly used in astronomy to refer to the Sun):

- $\boldsymbol{g_{\mathrm{M}} = 1.62\ \mathrm{m/s^2}}$
- $\boldsymbol{g_{\mathrm{V}} = 8.87\ \mathrm{m/s^2}}$
- $\boldsymbol{g_{\mathrm{J}} = 25.93\ \mathrm{m/s^2}}$
- $\boldsymbol{g_{\odot} = 273.77\ \mathrm{m/s^2}}$.

Problem 2

Geostationary satellites orbit the Earth in a stable orbit, so the force acting on them, i.e. the Earth's gravitational force, is the centripetal force:

$$G\frac{mM}{r^2} = m\omega^2 r = m\frac{4\pi^2}{T^2}r \quad \rightarrow \quad r = \sqrt[3]{GM\frac{T^2}{4\pi^2}} = 42227\ \mathrm{km}.$$

However, since the center of orbit is the center of the Earth, and not its surface, to obtain the distance from the surface, we need to subtract the radius of the Earth (6371 km). Thus, geostationary satellites orbit **35856 km** above the Earth's surface.

Problem 3

To determine the semi-major axis of Saturn's orbit, the most obvious solution is to use Kepler's Third Law. According to this law, the planets orbit the Sun in an elliptical orbit for which the relation between the semi-major axis and the orbital period is as follows:

$$\frac{a^3}{T^2} = \mathrm{constant}$$

Measuring the semi-major axis in astronomical units (AU) and the orbital period in years, the constant will be 1 (applied to the celestial bodies orbiting around the Sun), which is most easily seen by substituting the Earth's data. Rearranging the equation yields $a = T^{2/3}$. Plugging in the orbital period of Saturn (29.46 years) we reach the final result: $\mathbf{a_{Sat}} = \mathbf{9.54\ AU}$.

Problem 4

Let us use Kepler's Third Law the same way as in the previous problem: $T = a^{3/2} = 2536.7$ years. As the last perihelion of the comet was in 1997, the next perihelion is expected to occur in **4533**.

The perihelion distance can be obtained by subtracting the linear eccentricity (the distance between the center of the ellipse and its focal point, denoted by c) from the semi-major axis of the ellipse. The linear eccentricity can be calculated as the product of the semi-major axis and the (numerical) eccentricity e, so $c = ae = 185.07$ AU. hence, the perihelion distance is $\boldsymbol{r_p = a - c =} \mathbf{0.93\ AU}$.

Problem 5

Gravitational accceleration can be calculated the same way we did in the first problem: $g = GM/r^2$. To calculate the time of fall, let us use the equation of motion for uniformly accelerated motion:

$$s = \frac{1}{2}gt^2 = \frac{1}{2}\frac{GM}{r^2}t^2 \quad \rightarrow \quad t = \sqrt{\frac{2sr^2}{GM}}$$

Substituting the data gives that any object dropped from a height of 1 m (regardless of it is a stone or a feather) will fall for $\boldsymbol{t} = \mathbf{1.1\ s}$.